Closing Wed: HW_8 (8.3)
Closing Next Wed: HW_9A, 9B, 9C
Midterm 2 will be returned Tuesday.

9.1 Intro to Differential Equations
Goal: See a few differential equations and understand what a solution is.

A **differential equation** is an equation involving derivatives.

Notes:
$\frac{dy}{dt}$ = "instantaneous **rate of change** of *y* with respect to *t* "

"A is proportional to B" means
A = kB, where k is a constant.
In other words, A/B = k.

A survey of 4 applied examples:
1. Natural Unrestricted population
Assumption: "*The rate of growth of a population is proportional to the size of the population*."

P(t) = the population at year *t*,
$\frac{dP}{dt} =$ the rate of change of the population with respect to time (i.e. rate of growth).

So the assumption is equivalent to the differential equation

$$\frac{dP}{dt} = kP,$$

for some constant *k*
(we call *k* the *relative* *growth rate*)

2. Newton's Law of Cooling

Assumption: "*The rate of cooling is proportional to the temperature difference between the object and its surroundings*."

$T_s =$ constant temp. of the surroundings

$T(t) =$ the temp. of the object at time *t*,

$\frac{dT}{dt} =$ the rate of change of the temp with respect to time (*i.e.* rate of cooling).

$T - T_s =$ temp. difference between object and surroundings.

So Newton's Law of Cooling is equivalent to the differential equation

$$\frac{dT}{dt} = k(T - T_s),$$

for some constant *k* (cooling constant).

3. All motion problems!

Consider an object of mass *m* kg moving up and down on a straight line.

Let y(t) = `height at time *t*'

$$\frac{dy}{dt} = \text{`velocity at time } t\text{'}$$

$$\frac{d^2y}{dt^2} = \text{`acceleration at time } t\text{'}$$

Newton's 2nd Law says:

(mass)(acceleration) = Force

$$m\frac{d^2y}{dt^2} = \text{sum of forces on the object}$$

Only taking into account gravity:

$$m\frac{d^2y}{dt^2} = -mg$$

Consider gravity and air resistance (the force due to air resistance is proportional to velocity):

$$m\frac{d^2y}{dt^2} = -mg - k\frac{dy}{dt}$$

4. A Mixing Problem

Assume a 50 gallon vat is initially full of pure water. A salt water mixture is being dumped **into** the vat at 2 gal/min and this mixture contains 3 g/gal. The vat is mixed together.

At the same time, the mixture is coming **out** of the vat at 2 gal/min.

Let y(t) = grams of salt in vat at time t.

$\frac{y(t)}{50}$ = salt per gallon in vat at time, t.

$\frac{dy}{dt} =$ the rate (g/min) at which salt is changing with respect to time.

RATE IN = (3 g/gal)(2 gal/min) = 6 g/min

RATE OUT = ($\frac{y}{50}$ g/gal)(2gal/min) = $\frac{y}{25}$ g/min

Thus,

$$\frac{dy}{dt} = 6 - \frac{y}{25}$$

General Notes about Differential Equations

A **solution to a differential equation** is any function that satisfies the equation.

Consider the differential equation:

$$\frac{dP}{dt} = 2P$$

(a) P(t) = $8e^{2t}$ is a solution because

$\frac{dP}{dt} = 16e^{2t}$ and $2P = 16e^{2t}$.

(b) P(t) = t^3 is NOT a solution because

$\frac{dP}{dt} = 3t^2$ and $2P = 2t^3$.

(c) P(t) = 0 is a solution because

$\frac{dP}{dt} = 0$ and $2P = 0$.

(d) The **general solution** to

$$\frac{dP}{dt} = 2P$$

is

$$P(t) = Ce^{2t},$$

for any constant C.

We will learn how to find this next time.

Consider the differential equation:

$$y'' + 2y' + y = 0.$$

(a) Is $y = e^{-2t}$ a solution?

(b) Is $y = t\,e^{-t}$ a solution?

(c) There is a sol'n that looks like $y = e^{rt}$. Can you find the value of *r* that works?